Nutcracker: Problem set -1

Physics

Particle sliding on a cone A particle P slides on the smooth inner surface of a circular cone of semi-angle α. The axis of symmetry of the cone is vertical with the vertex O pointing downwards. Show that the vertical component of angular momentum about O is conserved in the motion. State a second dynamical quantity that is conserved. Initially P is a distance a from O when it is projected horizontally along the inside surface of the cone with speed u. Show that, in the subsequent motion, the distance r of P from O satisfies the equation

$$\dot{r}^2 = (r-a)\left[\frac{u^2}{r^2}(r+a) - 2gcos\alpha\right]$$

Case A For the case in which gravity is absent, find r and the azimuthal angle ϕ explicitly as functions of t. Make a sketch of the path of P (as seen from 'above') when $\alpha = \frac{\pi}{6}$.

Case B For the case in which $\alpha = \frac{\pi}{3}$, find the value of u such that r oscillates between a and $2a$ in the subsequent motion. With this value of u, show that r will first return to the value $r = a$ after a time

$$2\sqrt{3}\sqrt{\frac{a}{g}}\int \frac{\tau d\tau}{\sqrt{(\tau-1)(2-\tau)(2+3\tau)}}$$

Answers should be sent to one of the coordinators

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before 8:00 PM tomorrow(31^{st} august)